

# Sun-Earth L<sub>2</sub> Point Nulling Interferometry Formation Flying Performance Analysis

**Aiden Weatherbee**

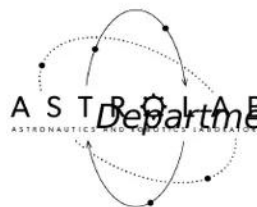
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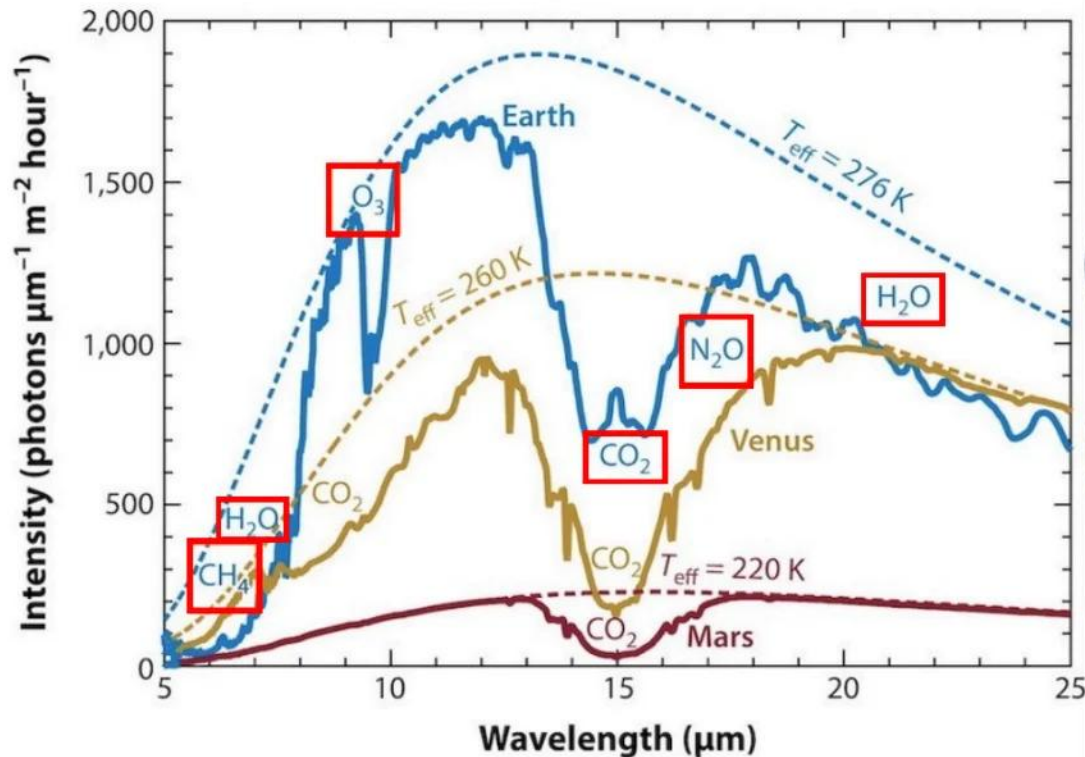
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# The Goal: Characterize Terrestrial Worlds with LIFE



Wavelength covers peak emission of Earth-like exoplanets



Features absorption bands of  $\text{O}_3$ ,  $\text{CH}_4$ ,  $\text{N}_2\text{O}$



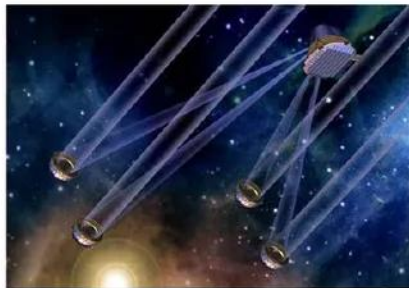
Thermal emission carries atmospheric information

For a nulling interferometer (e.g., LIFE) there are several mission level design variables that affect the design of both the formation control and the optics.

We study the relative formation error and fuel usage of these key variables.

**Formation control and optical control must be co-designed:** formation errors couple directly into nulling and modulation errors.

## Past Concepts:

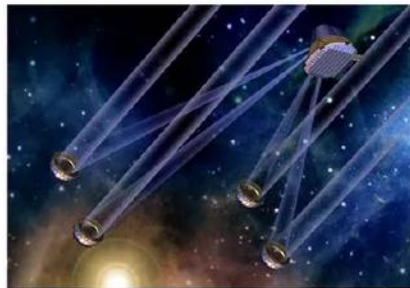


TPFI (credit: NASA)



DARWIN (credit: ESA)

## Past Concepts: Past/Future Tech Demos:



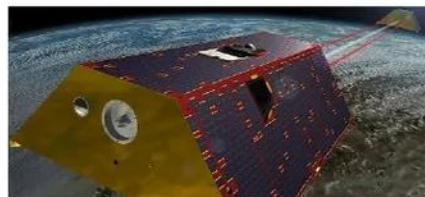
TPFI (credit: NASA)



DARWIN (credit: ESA)



Proba-3 (credit:



GRACE-FO (credit:



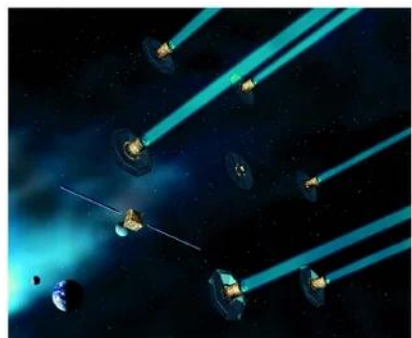
STARI (credit: Stanford  
SRI )



## Past Concepts: Past/Future Tech Demos: Inspiration in this



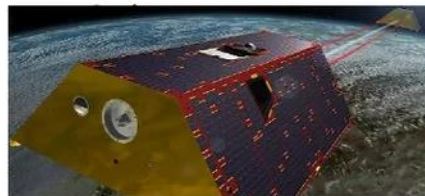
TPFI (credit: NASA)



DARWIN (credit: ESA)



Proba-3 (credit:



GRACE-FO (credit:

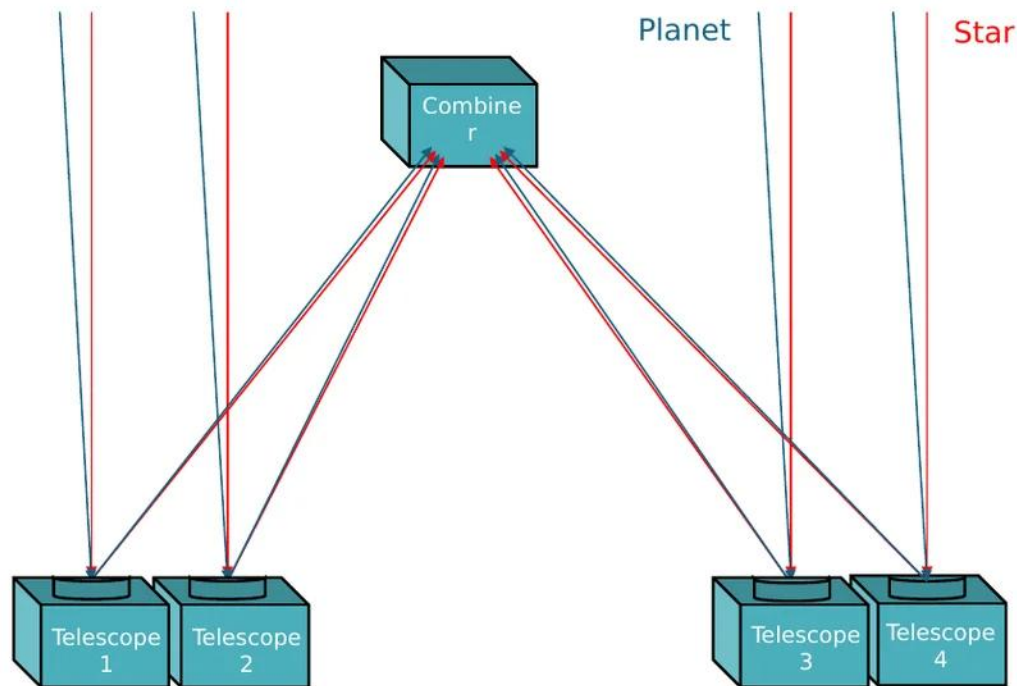


STARI (credit: Stanford SRI)

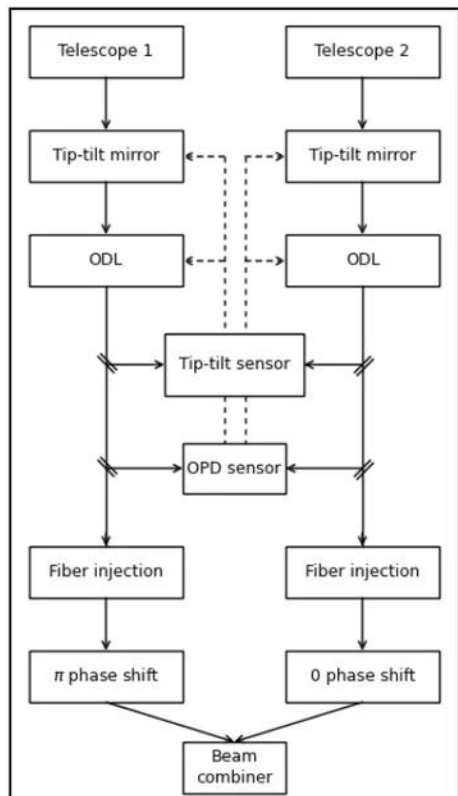
### Study:

- In-plane vs Out-of-plane (TPFI vs Darwin)
- 12, 18, 24 hour rotation rates (TPFI vs LIFE vs Darwin)
- 45, 65, 85 degree pointing angle (TPFI/LIFE/Darwin inspired)
- Sensors inspired by Proba-3, GRACE-FO, and STARI

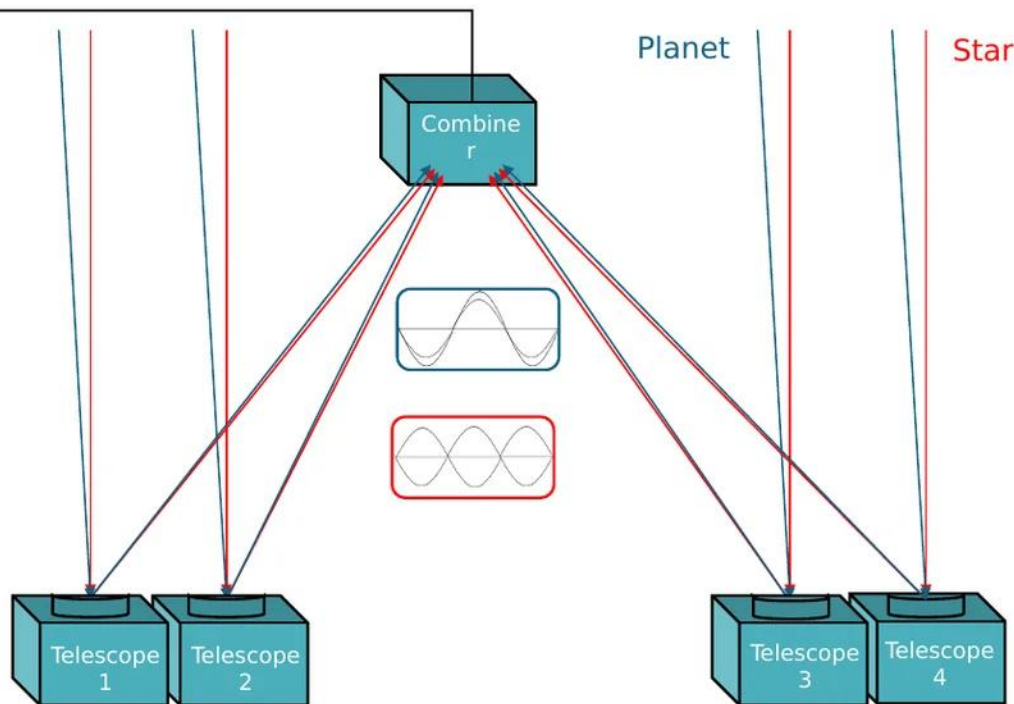
# Nulling Interferometry



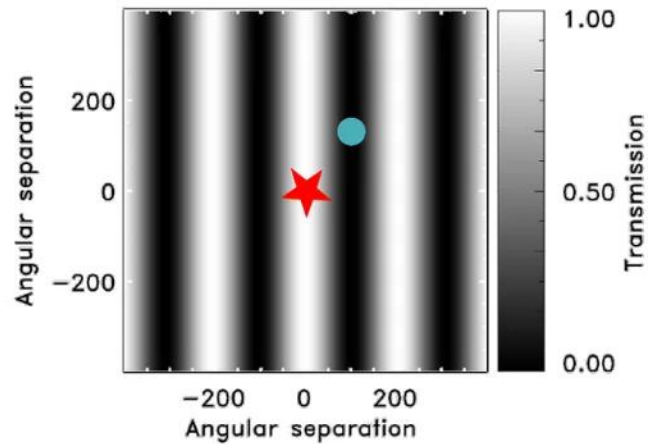
# Nulling Interferometry



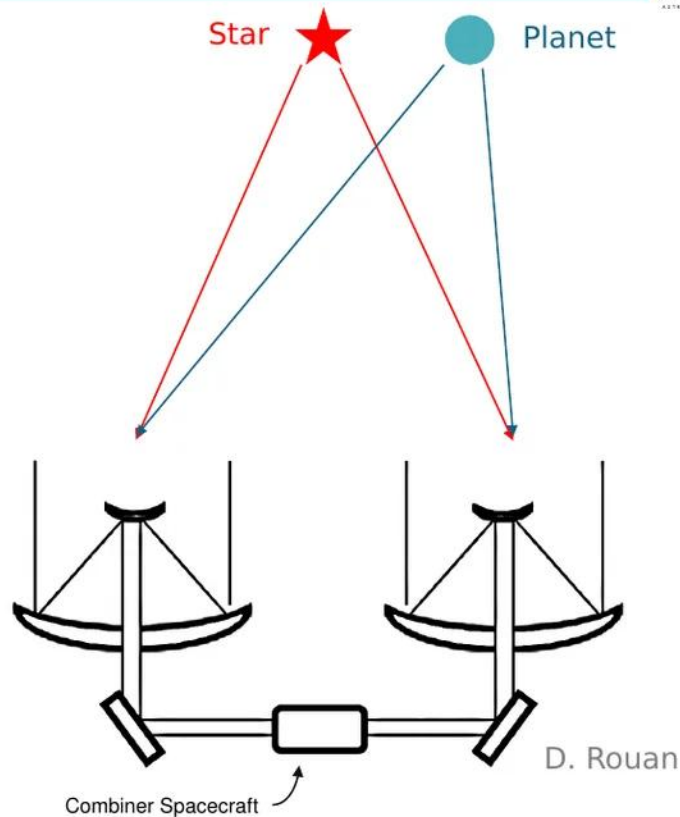
x2

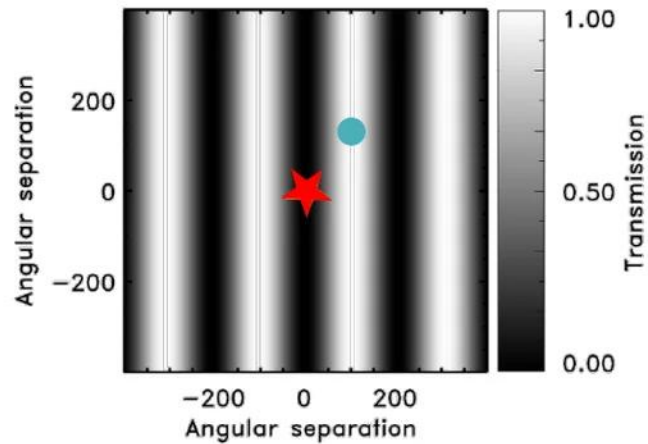




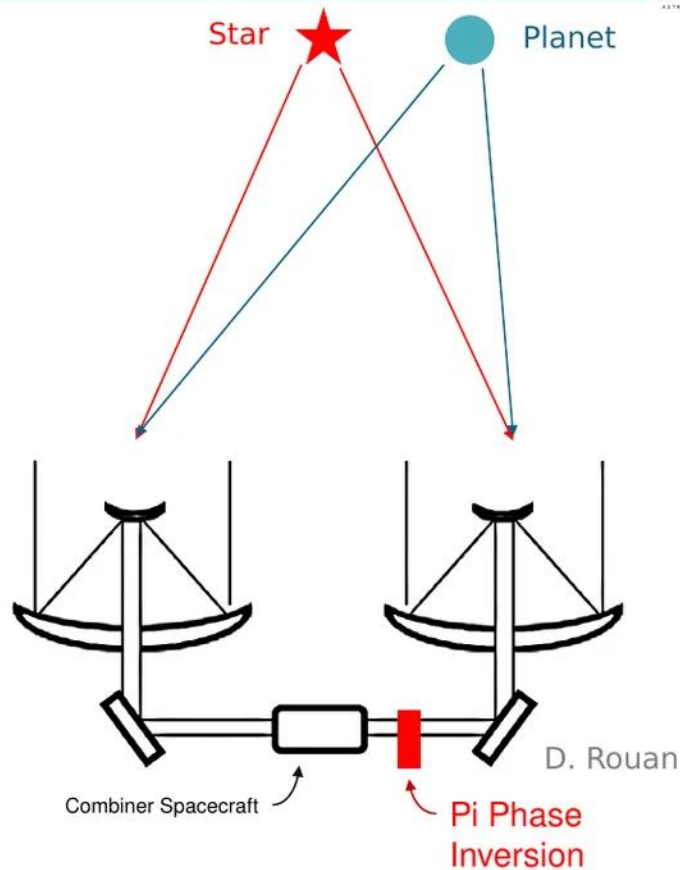


Imaging Interferometer





## Nulling Interferometer

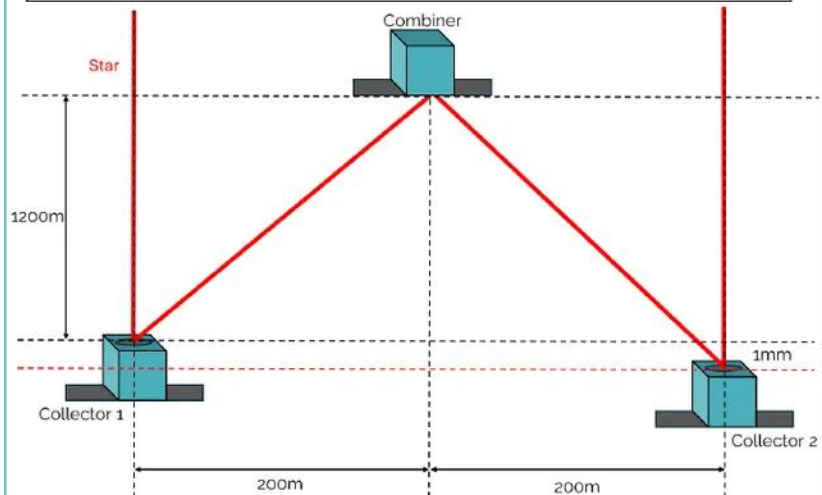


## Optical Path Difference / Phase Mismatch

$$\Delta \text{OPD}_{\star,ij} = -(\hat{\mathbf{s}}_{\star} + \hat{\boldsymbol{\rho}}_j)^{\top} \delta \mathbf{r}_j + (\hat{\mathbf{s}}_{\star} + \hat{\boldsymbol{\rho}}_i)^{\top} \delta \mathbf{r}_i$$

$$\hat{\boldsymbol{\rho}}_i = \frac{\mathbf{r}_C - \mathbf{r}_i}{\|\mathbf{r}_C - \mathbf{r}_i\|}$$

$$|\delta \text{OPD}_{\star}| = (1 \text{ mm}) \left( 1 + \frac{1200}{\sqrt{1200^2 + 200^2}} \right) = 1.986 \text{ mm}$$

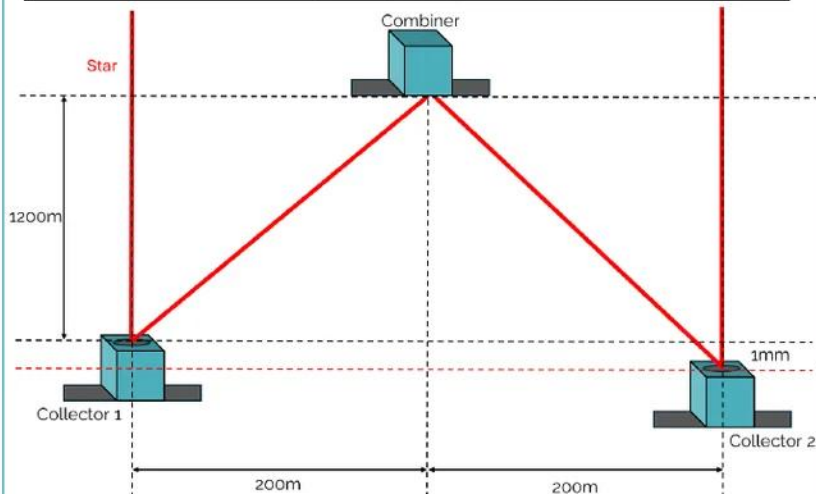


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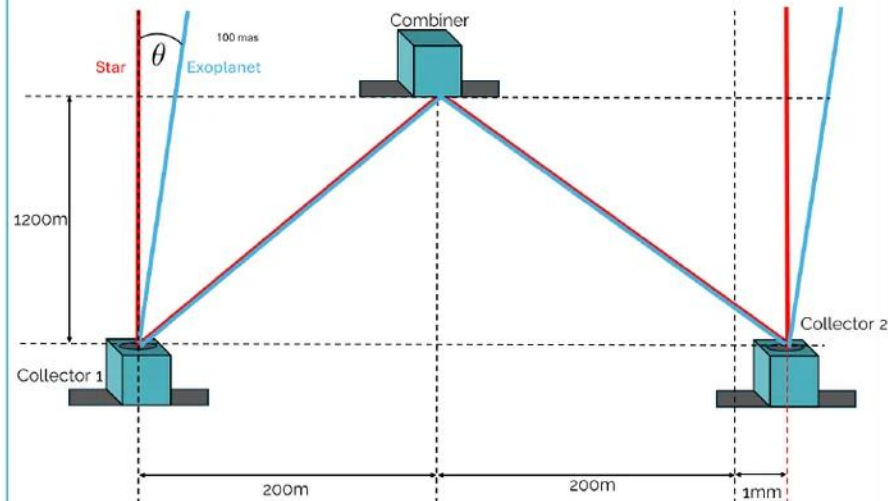
$$|\delta \text{OPD}_{\star}| = (1 \text{ mm}) \left( 1 + \frac{1200}{\sqrt{1200^2 + 200^2}} \right) = 1.986 \text{ mm}$$



## Planet Modulation Error

$$\delta \ell_{p-\star} = \delta \mathbf{b}_{ij}^{\top} (\hat{\mathbf{s}}_p - \hat{\mathbf{s}}_{\star}) \simeq \delta b_{\perp} \theta$$

$$|\delta D_{p-\star}| \simeq \delta b_{\perp} \theta = (10^{-3} \text{ m}) (4.848 \times 10^{-7}) = 0.485 \text{ nm}$$



# 144 Case Trade Study Variables

**2 x Controller**

PD vs CR3BP-LQI



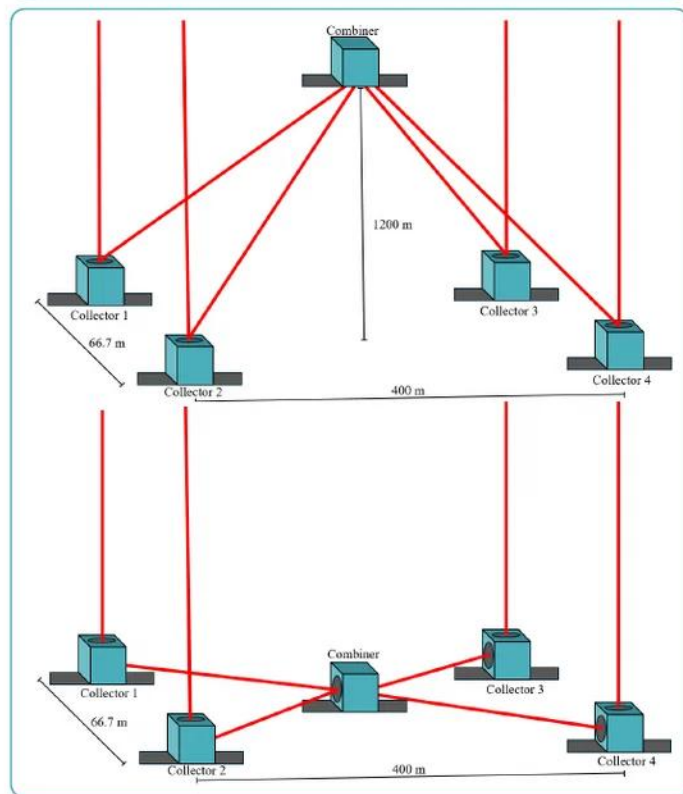
# 144 Case Trade Study Variables

**2 x Controller**

PD vs CR3BP-LQI

**2 x Combiner placement**

In-plane combiner vs out-of-plane combiner



# 144 Case Trade Study Variables

|                               |                                            |
|-------------------------------|--------------------------------------------|
| <b>2 x Controller</b>         | PD vs CR3BP-LQI                            |
| <b>2 x Combiner placement</b> | In-plane combiner vs out-of-plane combiner |
| <b>3 x Array rotation</b>     | 12 h, 18 h, 24 h array-rotation periods    |



# 144 Case Trade Study Variables

**2 x Controller**

PD vs CR3BP-LQI

**2 x Combiner placement**

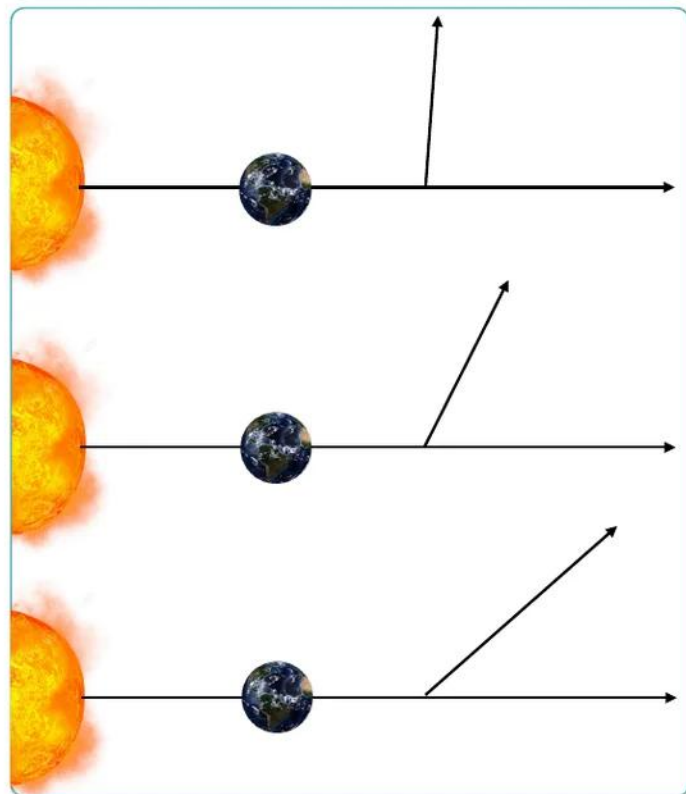
In-plane combiner vs out-of-plane combiner

**3 x Array rotation**

12 h, 18 h, 24 h array-rotation periods

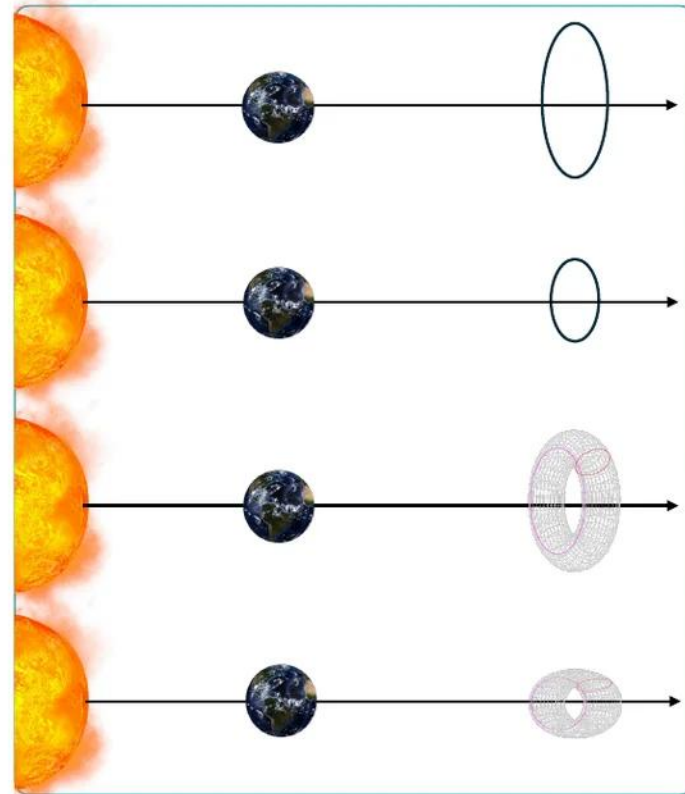
**3 x Pointing angle**

45°, 65°, 85° line-of-sight cases



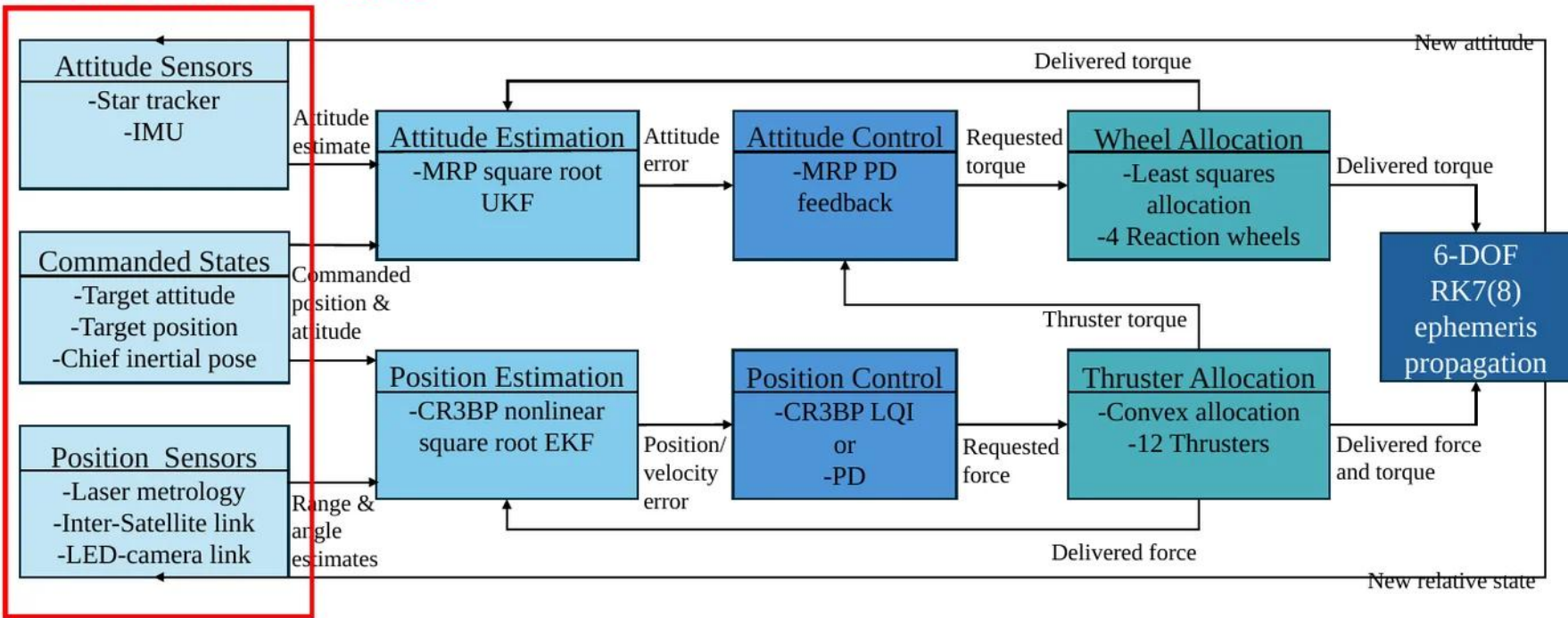
# 144 Case Trade Study Variables

|                               |                                                                              |
|-------------------------------|------------------------------------------------------------------------------|
| <b>2 x Controller</b>         | PD vs CR3BP-LQI                                                              |
| <b>2 x Combiner placement</b> | In-plane combiner vs out-of-plane combiner                                   |
| <b>3 x Array rotation</b>     | 12 h, 18 h, 24 h array-rotation periods                                      |
| <b>3 x Pointing angle</b>     | 45°, 65°, 85° line-of-sight cases                                            |
| <b>4 x Combiner orbit</b>     | JWST-like halo; small periodic halo; quasi-halo; Lissajous-family trajectory |

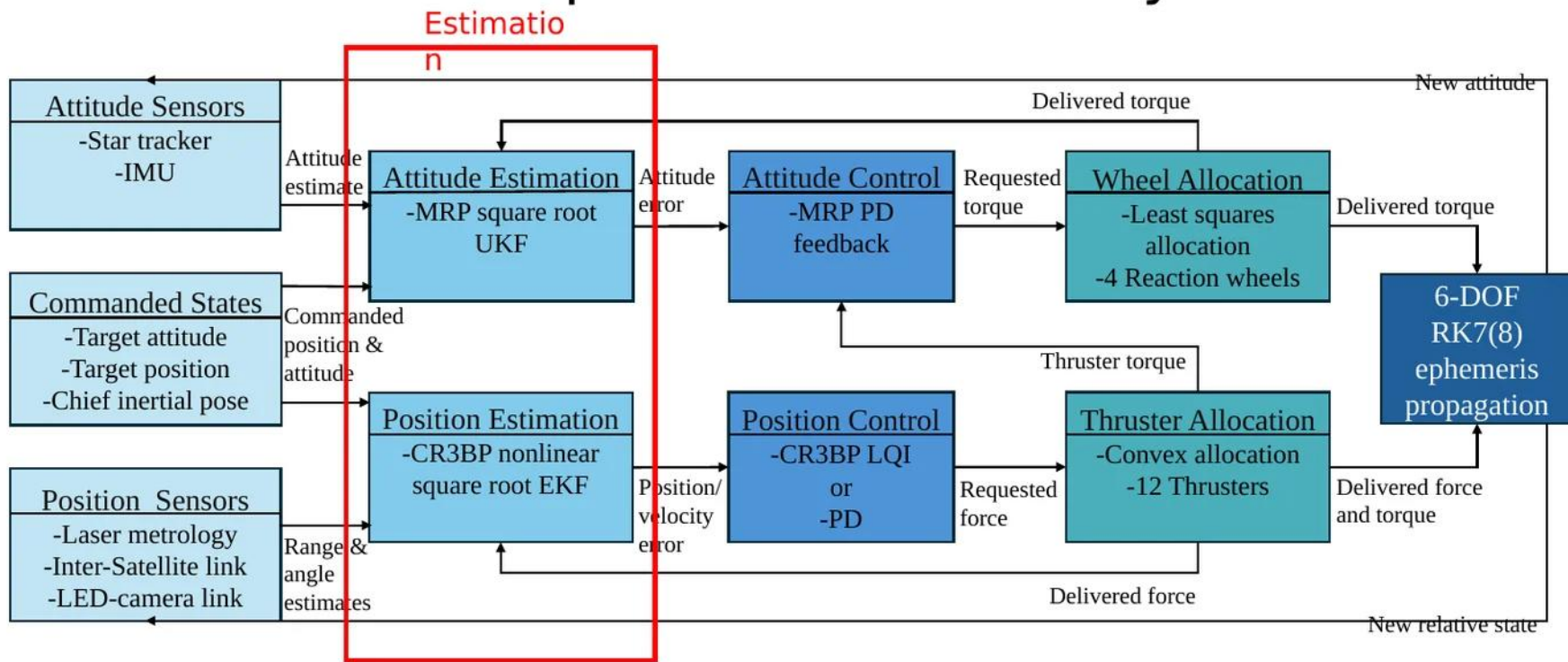


# Follower Spacecraft Control System

## Sensors/Commanded states

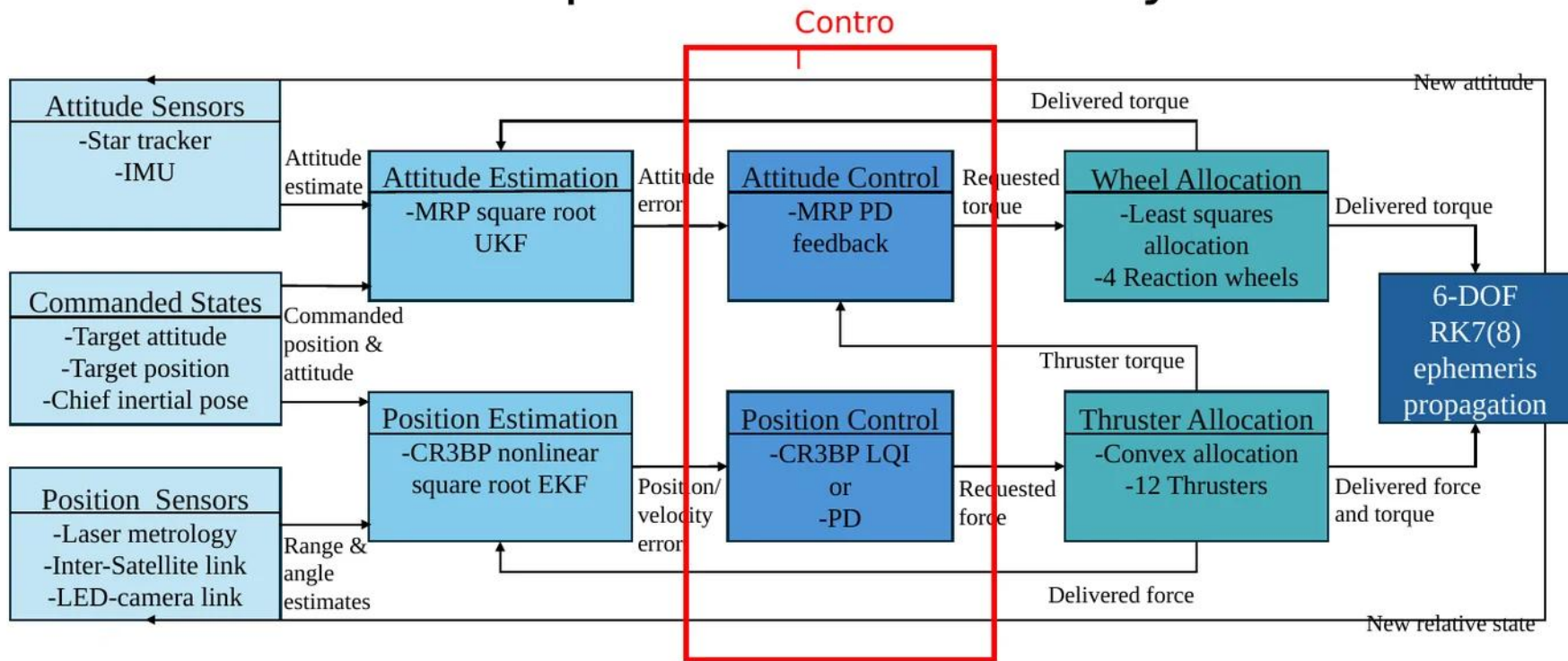


# Follower Spacecraft Control System

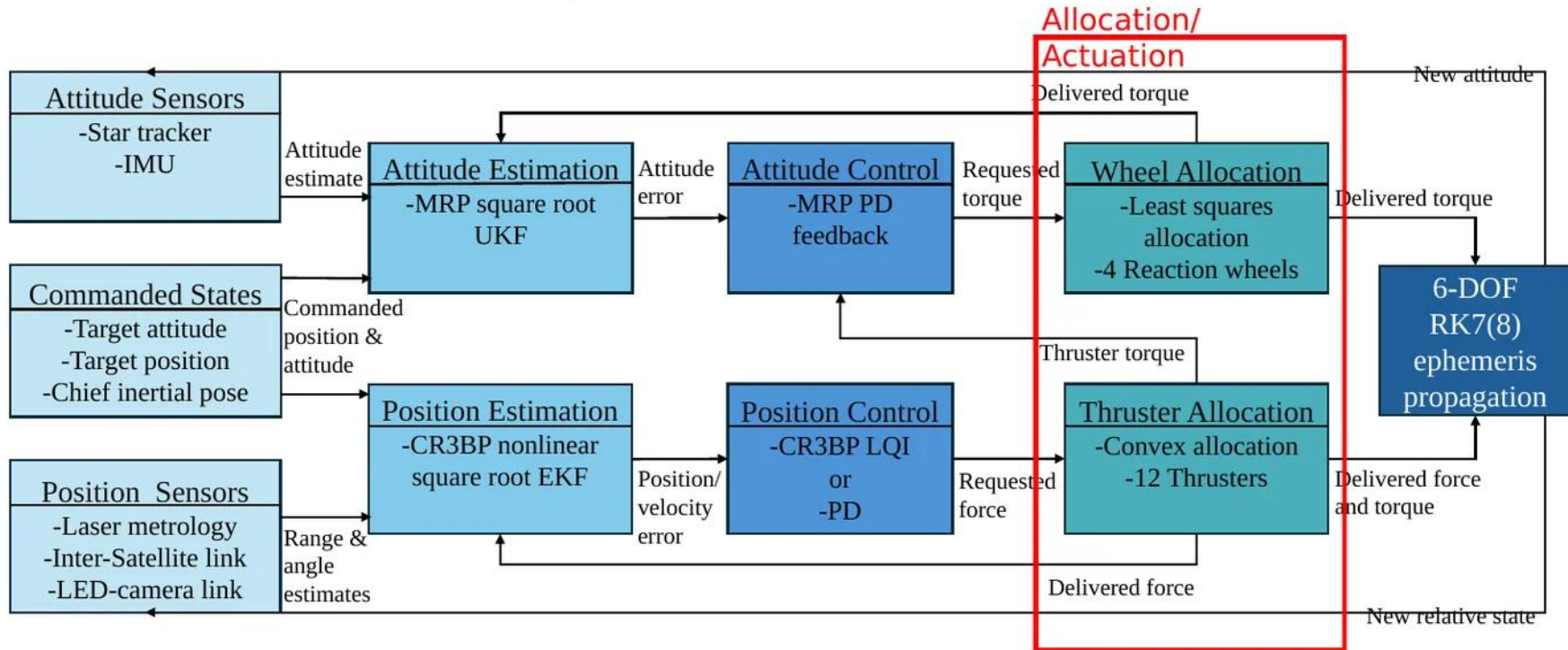




# Follower Spacecraft Control System

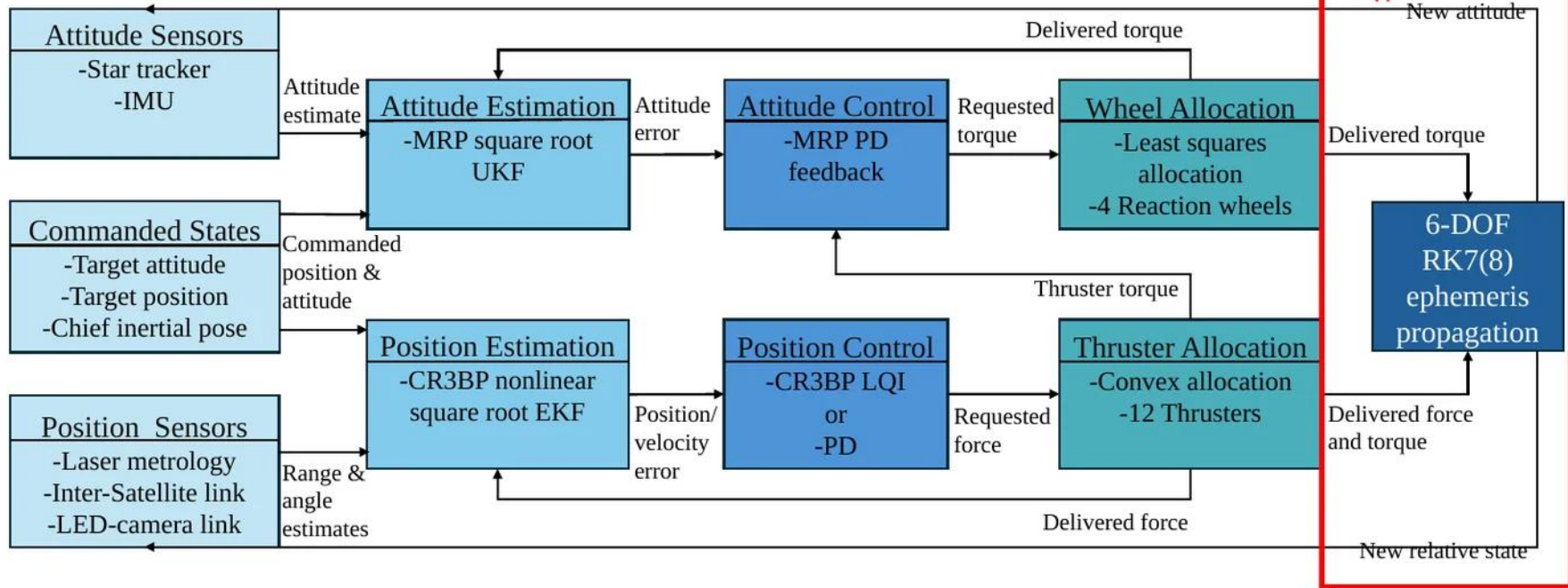


# Follower Spacecraft Control System



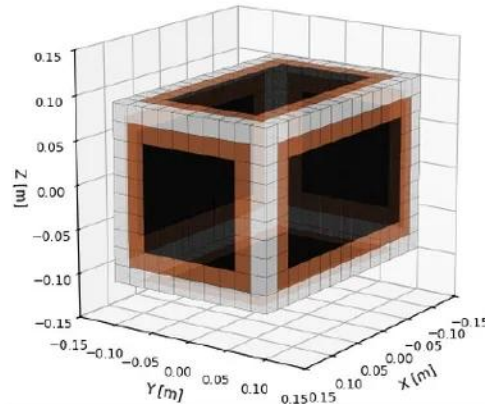
# Follower Spacecraft Control System

Propagation  
n



$$m\ddot{\mathbf{r}}_N = \mathbf{F}_{\text{grav}} + \mathbf{F}_{\text{SRP}} + \mathbf{F}_{\text{th}} + \mathbf{F}_{\text{alb/IR}}$$

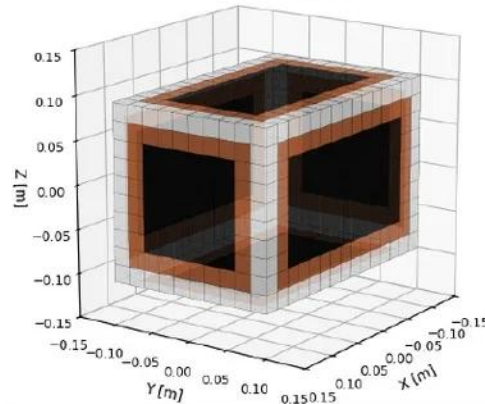
$$\mathbf{I}_B \dot{\boldsymbol{\omega}}_{BN,B} + \boldsymbol{\omega}_{BN,B} \times (\mathbf{I}_B \boldsymbol{\omega}_{BN,B}) = \boldsymbol{\tau}_{\text{gg}} + \boldsymbol{\tau}_{\text{SRP}} + \boldsymbol{\tau}_{\text{th}} + \boldsymbol{\tau}_{\text{alb/IR}}$$



Each surface area is assigned, normal direction, reflectivity, absorptivity, emissivity, and thermal reradiation. Forces and torques are summed per surface.

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## Gravitational force and torque

 $\mathbf{F}_{\text{grav}}$ 
 $\boldsymbol{\tau}_{\text{gg}}$ 

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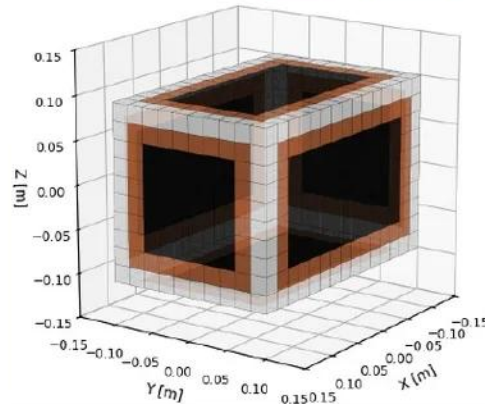


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## Solar radiation force and torque

$$\mathbf{F}_{\text{SRP}} \quad \boldsymbol{\tau}_{\text{SRP}}$$



## Gravitational force and torque

$$\mathbf{F}_{\text{grav}} \quad \boldsymbol{\tau}_{\text{gg}}$$

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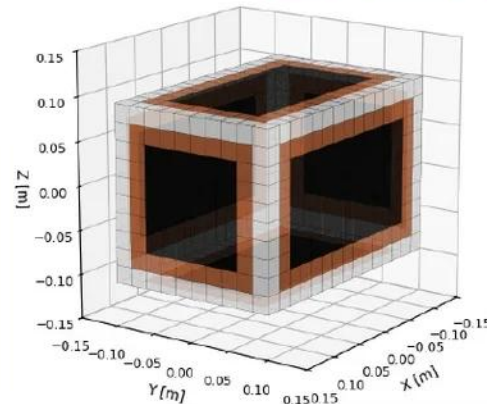


$$m\ddot{\mathbf{r}}_N = \mathbf{F}_{\text{grav}} + \mathbf{F}_{\text{SRP}} + \mathbf{F}_{\text{th}} + \mathbf{F}_{\text{alb/IR}}$$

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### Solar radiation force and torque

$$\mathbf{F}_{\text{SRP}} \quad \boldsymbol{\tau}_{\text{SRP}}$$



### Gravitational force and torque

$$\mathbf{F}_{\text{grav}} \quad \boldsymbol{\tau}_{\text{gg}}$$

### Thermal reradiation force and torque

$$\mathbf{F}_{\text{th}} \quad \boldsymbol{\tau}_{\text{th}}$$

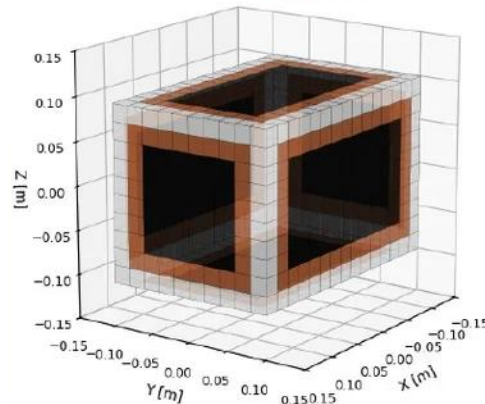
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### Solar radiation force and torque

$$\mathbf{F}_{\text{SRP}} \quad \boldsymbol{\tau}_{\text{SRP}}$$



### Gravitational force and torque

$$\mathbf{F}_{\text{grav}} \quad \boldsymbol{\tau}_{\text{gg}}$$

### Thermal reradiation force and torque

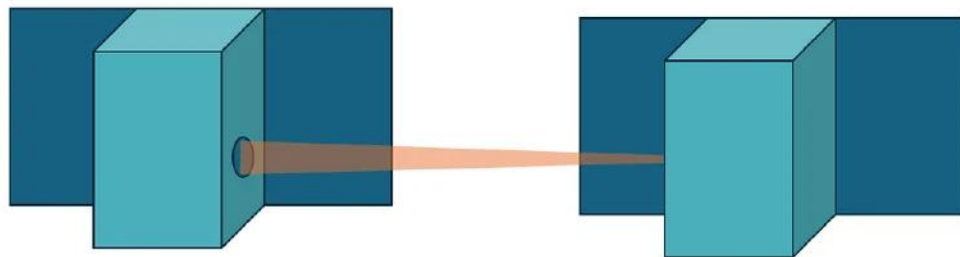
$$\mathbf{F}_{\text{th}} \quad \boldsymbol{\tau}_{\text{th}}$$

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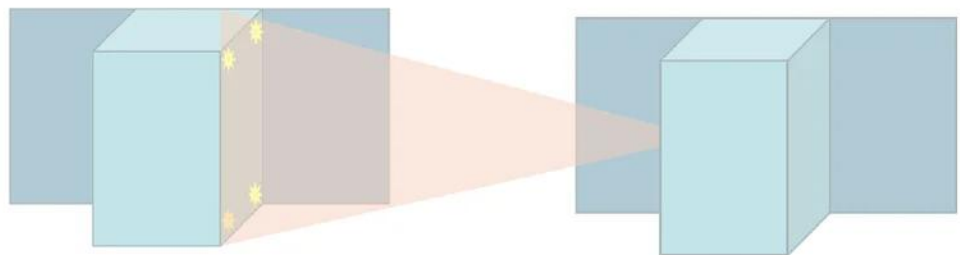
### Earth albedo and infrared light force and torque

$$\mathbf{F}_{\text{alb/IR}} \quad \boldsymbol{\tau}_{\text{alb/IR}}$$

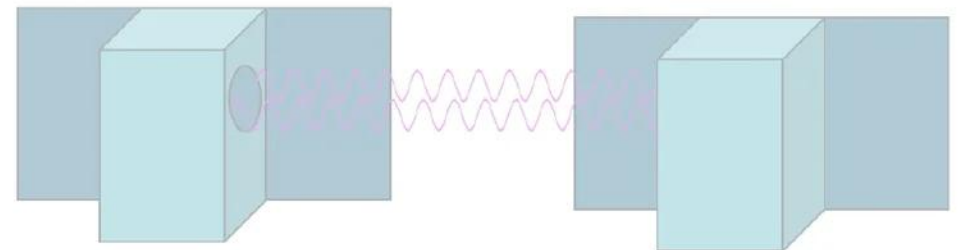
# Inter-Satellite Sensors



Laser Metrology



LED Centroiding



Inter-Satellite  
Link

# Laser Metrology

$$\rho_{\text{meas}} = \rho_{\text{true}} + \delta\rho_{\text{shot}} + \delta\rho_{\text{freq}} + \delta\rho_{\text{TTL}} + \delta\rho_{\text{thermal}}$$

 $\delta\rho_{\text{shot}}$ 

Radiometric link budget with a **white noise draw** that scales inversely with the square root of that received power.

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**Pink noise signal** via a first-order autoregressive process, scaled proportionally to the distance between spacecraft.

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The **Path change caused by spacecraft pointing jitter** is added to a randomized linear coupling matrix representing lingering post-calibration errors.



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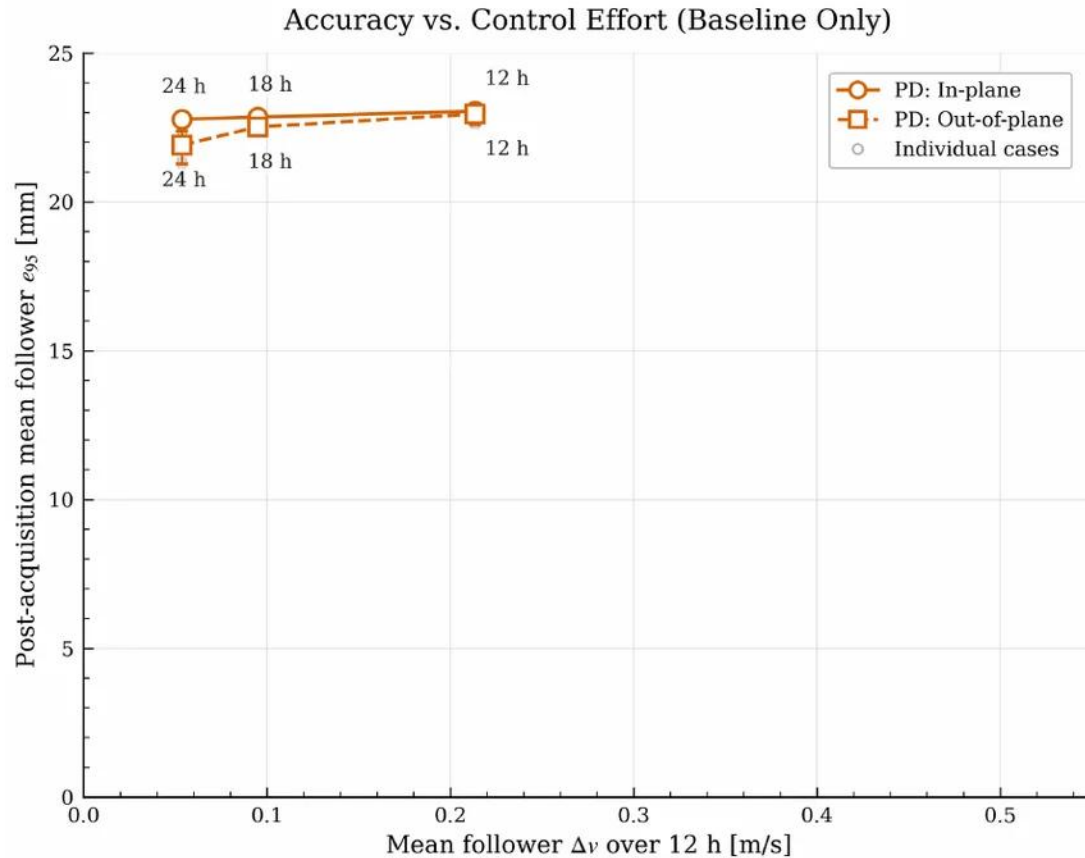
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 $\delta\rho_{\text{TTL}}$ 

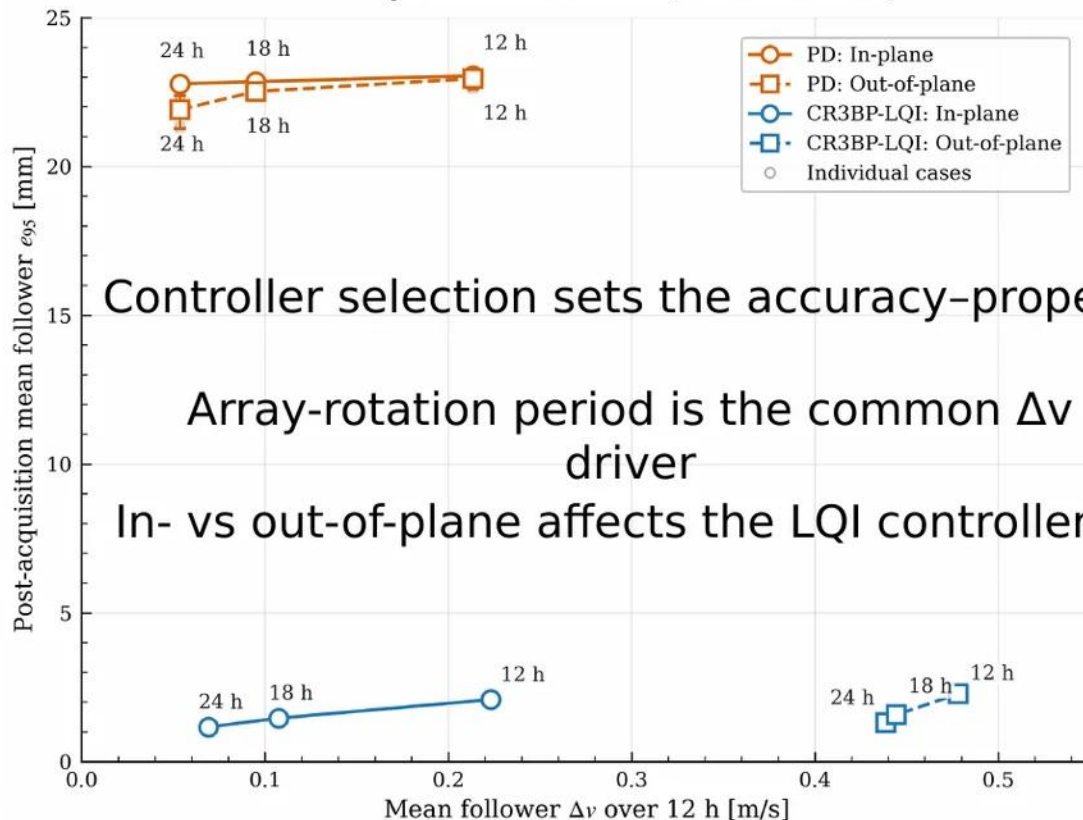
The **Path change caused by spacecraft pointing jitter** is added to a randomized linear coupling matrix representing lingering post-calibration errors.

 $\delta\rho_{\text{thermal}}$ 

Simulated via a **first-order Gauss-Markov random walk** applied directly to the body-frame metrology head mounting vectors.



Accuracy vs. Control Effort (All Controllers)



Controller selection sets the accuracy-propellant trade

Array-rotation period is the common  $\Delta v$  driver

In- vs out-of-plane affects the LQI controller only

# Relative Factor Sensitivity Percentages

Formation error

Control effort

|                    | Formation error       |                | Control effort          |                  |
|--------------------|-----------------------|----------------|-------------------------|------------------|
| Rotation period    | 57.3                  | 2.9            | 33.0                    | 132.6            |
| Chief placement    | 9.2                   | 1.9            | 109.2                   | 0.0              |
| Pointing angle     | 4.5                   | 2.1            | 0.4                     | 0.1              |
| Chief orbit family | 0.2                   | 0.1            | 0.2                     | 0.0              |
|                    | CR3BP-LQI<br>$e_{95}$ | PD<br>$e_{95}$ | CR3BP-LQI<br>$\Delta v$ | PD<br>$\Delta v$ |

## Key Takeaways

**GNC and optical control must be co-designed:** Formation errors couple directly into optical-path difference and planet-modulation error.

**Controller selection sets the accuracy-propellant trade:** CR3BP-LQI achieved 1-2 mm error, but came with the cost of higher  $\Delta v$ ; with LQI, the out-of-plane combiner adds a large  $\Delta v$  penalty.

**Array-rotation periods are the common  $\Delta v$  driver:** Array rotation periods raise  $\Delta v$ , and substantially increase LQI formation error. Pointing angle and chief-orbit family were secondary.

## Any Questions?

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York University, Email: [aidenweatherbee@hotmail.com](mailto:aidenweatherbee@hotmail.com)*